RESEARCH ARTICLE

Some Results on Fuzzy Supra Baire Spaces and Fuzzy Supra Strongly Hyperconnected Spaces

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ABSTRACT: The purpose of this article is to introduce some results on fuzzy supra Baire spaces and other fuzzy supra topological spaces. The characterization and relation between fuzzy supra strongly hyperconnected spaces and fuzzy supra Baire spaces were carried out. From the results, we have shown that fuzzy supra strongly hyperconnected spaces which are not fuzzy supra Baire spaces. In addition, fuzzy supra strongly hyperconnected spaces are fuzzy supra nodec space, fuzzy supra hyperconnected space and fuzzy supra open hereditarily irresolvable space were discussed.

KEYWORDS: Fuzzy Supra Baire Space, Fuzzy Supra nodec space, Fuzzy Supra irresolvable space, Fuzzy Supra open hereditarily irresolvable Space, Fuzzy Supra Strongly Hyperconnected

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1. INTRODUCTION

In the new approach of set theory in the name of fuzzy sets concepts was introduced by Zadeh [1] in 1965. The notion has been successfully applied in all branches of Mathematics. Later, In 1968, the concepts of fuzzy topological space was introduced and developed by Chang [2]. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In 1987, the concept of fuzzy supra topological spaces was introduced by Abd El-Monsef et.al [3]. The aim of this article is to discuss some results on fuzzy supra Baire space and other fuzzy supra topological spaces. Also finding the characterizations of fuzzy supra strongly hyperconnected spaces are obtained.

2. PRELIMINARIES

Definition 2.1 [4]

In a fuzzy supra topological space (X,T^*), a fuzzy set λ is said to be a fuzzy supra first category set if $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are fuzzy supra nowhere dense sets in (X,T^*). Any other fuzzy set in (X,T^*) is said to be a fuzzy supra second category set in (X,T^*).

Definition 2.2 [5]

In a fuzzy supra topological space (X,T^*), a fuzzy set λ is said to be fuzzy supra strongly nowhere dense set, if $\lambda \wedge (1-\lambda)$ is a fuzzy supra nowhere dense set in (X,T^*). That is $int^*\{cl^*[\lambda \wedge (1-\lambda)]\} = 0$, in (X,T^*).

Definition 2.3 [5]

In a fuzzy supra topological space (X, T^*) is called a fuzzy supra strongly Baire space if $cl^*(\wedge_{i=1}^{\infty}(\lambda_i))=1$, where (λ_i)'s are fuzzy supra strongly nowhere dense sets in (X,T^*).

Definition 2.4 [6]

In a fuzzy supra topological space (X,T^*) is called a fuzzy supra submaximal space if for each fuzzy set in (X,T^*) such that $cl^*(\lambda) = 1$, then $\lambda \epsilon T^*$ in (X, T^*).

Theorem 2.1 [4]

Let (X,T^*) be a fuzzy supra topological space. Then the following are equivalent:

(i) (X,T^*) is a fuzzy supra Baire space.

(ii) $int^*(\lambda) = 0$, for every fuzzy supra first category set λ in (X,T^*).

(iii) $cl^*(\mu) = 1$, for every fuzzy supra residual set μ in (X,T^*).

Theorem 2.2 [5]

If (X, T^*) is a fuzzy supra strongly hyper connected space, then

(i) (X, T^*) is a fuzzy supra hyper connected space,

(ii) (X, T^*) is a fuzzy supra submaximal space.

Theorem 2.3 [5]

If $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy supra

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dense sets in a fuzzy supra strongly hyper connected space (X, T*), then $1 - \lambda$ is a fuzzy supra first category set in (X,T*).

Theorem 2.4 [5]

If λ and μ are any two non-zero fuzzy supra open sets in a fuzzy supra strongly hyperconnected space (X, T*), then $\lambda \wedge \mu \neq 0$, in (X, T*).

Theorem 2.5 [5]

If $int^*(\lambda)=0$ and $int^*(\mu)=0$ for any two non-zero fuzzy sets defined on X in a fuzzy supra strongly hyperconnected space (X, T*) then $int^*(\lambda \vee \mu) = 0$, in (X,T*).

Definition 2.5 [6]

In a fuzzy supra topological space (X,T*) is called a fuzzy supra extremally disconnected space if $cl^*(\lambda)= \lambda$, where $\lambda \in T^*$.

Theorem 2.6 [6]

Every fuzzy supra hyperconnected space (X,T*) is a fuzzy supra extremally disconnected space.

3. Characterizations of Fuzzy Supra Spaces

Definition 3.1

Let (X,T*) be a fuzzy supra topological space. A fuzzy set λ defined on X is called a fuzzy supra somewhere dense set, if $int^*cl^*(\lambda) \neq 0$ in (X,T*). That is, λ is a fuzzy supra somewhere dense set in (X,T*) if there exists a non-zero fuzzy supra open set μ in (X,T*) such that $\mu \leq cl^*(\lambda)$.

Definition 3.2

A fuzzy supra topological space (X,T*) is called a fuzzy supra open hereditarily irresolvable space if $int^*[cl^*(\lambda)] \neq 0$, for any non - zero fuzzy set λ in (X,T*).

Proposition 3.1

If λ is a fuzzy set defined on X in a fuzzy supra open hereditarily irresolvable space (X,T*), then $1-\lambda$ is a fuzzy supra somewhere dense set in (X,T*)

Proof:

Let λ be a fuzzy set in (X,T*). Then, $int^*(\lambda)$ is a fuzzy supra open set in (X,T*). Since (X,T*) is a fuzzy supra open hereditarily irresolvable space. Therefore, $int^*(\lambda)$ is not a fuzzy supra dense set in (X,T*). That is, $cl^*int^*(\lambda) \neq 1$ and then $1 - cl^*int^*(\lambda) \neq 0$.This implies that $int^*cl^*(1-\lambda) \neq 0$. Hence $1-\lambda$ is a fuzzy supra somewhere dense set in (X,T*).

Proposition 3.2

If (X,T*) is a fuzzy supra open hereditarily irresolvable space, then $int^*(\lambda) \neq 0$, for any non–zero fuzzy set λ in (X,T*), implies that $int^*cl^*(\lambda)\neq 0$.

Proof:

Let (X, T*) be a fuzzy supra open hereditarily irresolvable space and λ be a fuzzy set defined on (X,T*), such that $int^*cl^*(\lambda) \neq 0$, in (X,T*). Suppose that $int^*(\lambda) = 0$, in (X,T*). Then, $cl^*(1-\lambda) = 1-int^*(\lambda) = 1-0=1$ and then $1-\lambda$ will be a fuzzy supra dense set in the fuzzy supra open hereditarily irresolvable space (X,T*), by Proposition 3.1, it is a contradiction to $1-\lambda$ being a fuzzy supra somewhere dense set in (X,T*), and thus if $int^*cl^*(\lambda) \neq 0$, for a non-zero fuzzy set λ in a fuzzy supra open hereditarily irresolvable space, then $int^*(\lambda) \neq 0$, in (X,T*).

Definition 3.3

A fuzzy supra topological space (X,T*) is called a fuzzy supra perfectly disconnected space, if for any two non-zero fuzzy sets λ and μ defined on X with $\lambda \leq 1 - \mu$, $cl^*(\lambda) \leq 1 - cl^*(\mu)$ in (X,T*).

Proposition 3.3

If (X,T*) is a fuzzy supra perfectly disconnected space, then (X,T*) is a fuzzy supra extremally disconnected space.

Proof:

Let λ be a fuzzy set defined on (X,T*). Since (X,T*) is a fuzzy supra perfectly disconnected space and $(1 - \lambda) \leq 1 - [int^*(\lambda)]$ in (X,T*) implies that $cl^*(1-\lambda) \leq 1 - cl^*[int^*(\lambda)]$ and then $1 - int^*(\lambda) \leq 1 - cl^*int^*(\lambda)$ and hence $cl^*int^*(\lambda) \leq int^*(\lambda)$. But $int^*(\lambda) \leq cl^*[int^*(\lambda)]$ in (X,T*). Hence, $cl^*int^*(\lambda) = int^*(\lambda)$, in (X,T*). Let $\mu= int^*(\lambda)$, then μ is a fuzzy open set in (X,T*). Now $cl^*(\mu) = \mu$ and $\mu \in T^*$ implies that $cl^*(\mu) \in T^*$. Hence, if $\mu \in T^*$ then $cl^*(\mu) \in T^*$ implies that (X,T*) is a fuzzy supra extremally disconnected space.

Definition 3.4

A fuzzy supra topological space (X,T*) is called a fuzzy supra nodec space, if every non-zero fuzzy supra nowhere dense set is fuzzy supra closed in (X,T*). That is., if λ is a fuzzy supra nowhere dense set in (X,T*), then $1-\lambda \in T^*$.

Proposition 3.4

A fuzzy supra topological space (X,T*) is a fuzzy supra submaximal space if and only if (X,T*) is a fuzzy supra nodec and fuzzy supra open hereditarily irresolvable space.

Proof:

Let λ be a fuzzy supra dense set in (X,T*). Then $cl^*(\lambda) = 1$, in (X,T*). Now $int^*(1-\lambda) = 1 - cl^*(\lambda) = 0$, in (X,T*). Since (X,T*) is a fuzzy supra open hereditarily irresolvable space, $int^*(1-\lambda)=0$, in (X,T*) implies that $int^*cl^*(1-\lambda)=0$. Then $(1-\lambda)$ is a fuzzy supra nowhere dense set in (X,T*). Also, since (X,T*) is a fuzzy supra nodec space, the fuzzy supra nowhere dense set, $(1-\lambda)$ is a fuzzy supra closed set in (X,T*). Then λ is a fuzzy supra open set in (X,T*). Hence, the fuzzy supra dense set λ is a fuzzy supra open set in (X,T*). Therefore, (X,T*) is a fuzzy supra submaximal space.

Conversely, Let (X,T*) be a fuzzy supra submaximal space and let μ be a fuzzy supra nowhere dense set in (X,T*). Then $int^*cl^*(\mu)=0$ in (X,T*). But $int^*(\mu) \leq int^*cl^*(\mu)$ in (X,T*) implies that $int^*(\mu) \leq 0$ in (X,T*). That is., $int^*(\mu)=0$ in (X,T*). Now $cl^*(1-\mu)= 1-int^*(\mu)=1-0=1$ in (X,T*). Then $(1-\mu)$ is a fuzzy supra dense set in (X,T*). Since (X,T*) is a fuzzy supra submaximal space, the fuzzy supra dense set $(1-\mu)$ is a fuzzy supra open set in (X,T*). Then, μ is a fuzzy supra closed set in (X,T*). Hence the fuzzy supra nowhere dense set μ is a fuzzy supra closed set in (X,T*). Therefore (X,T*) is a fuzzy supra nodec space.

Let δ be a fuzzy supra somewhere dense set in (X,T*). Then, $int^*[cl^*(\delta)] \neq 0$ in (X,T*). It has to be claimed that $int^*(\delta)\neq 0$ in (X,T*). Assume the contrary. Suppose that $int^*(\delta)=0$ in (X,T*). Then, $cl^*(1-\delta)=1-int^*(\delta)=1-0=1$ in (X,T*), and hence $1-\delta$ will be a fuzzy supra dense set in (X,T*). Since (X,T*) is a fuzzy supra submaximal space, $1-\delta$ will be a fuzzy supra open set in (X,T*) and then δ is a fuzzy supra closed set in (X,T*) and $cl^*(\delta) = \delta$ in (X,T*) and then $int^*[cl^*(\delta)] = int^*(\delta)=0$, in (X,T*). This shows that $int^*[cl^*(\delta)] = 0$ in (X,T*), a contradiction to $int^*cl^*(\delta)\neq 0$, in (X,T*). Therefore $int^*(\delta)\neq 0$ in (X,T*). Hence for each fuzzy supra somewhere dense set δ in (X,T*), $int^*(\delta)\neq 0$ in (X,T*). Therefore, (X,T*) is a fuzzy supra open hereditarily irresolvable space.

Definition 3.5

A fuzzy supra topological space (X,T*) is called a fuzzy supra almost P-space if for every non-zero fuzzy supra G_δ-set λ in (X,T*), $int^*(\lambda) \neq 0$, in (X,T*).

Definition 3.6

A fuzzy supra topological space (X,T*) is called a fuzzy supra almost GP-space if $int^*(\lambda) \neq 0$, for each non – zero fuzzy supra dense and fuzzy supra G_δ-set λ in (X,T*). That is., (X,T*) is a fuzzy supra almost GP-space, if for each non - zero fuzzy supra G_δ- set λ in (X,T*), with $cl^*(\lambda) = 1$, $int^*(\lambda) \neq 0$.

Proposition 3.5

If λ is a fuzzy supra F_σ-set in a fuzzy supra almost P- space (X,T*), then $cl^*(\lambda) \neq 1$.

Proof:

Let λ be a fuzzy supra F_σ-set in a fuzzy supra almost P- space (X,T*). Then,$(1-\lambda)$ is a fuzzy supra G_δ-set in (X,T*). Since (X,T*) is a fuzzy supra almost P-space, for the fuzzy supra G_δ-set $(1-\lambda)\neq 0$ we have $int^*(1-\lambda)\neq 0$. This implies that $1-cl^*(\lambda)\neq 0$ and hence we have $cl^*(\lambda) \neq 1$.

Proposition 3.6

If a fuzzy supra topological space (X,T*) is a fuzzy supra almost P-space, then (X,T*) is a fuzzy supra second category space.

Proof:

Let the fuzzy supra topological space (X,T*) be a fuzzy supra almost P-space. Suppose that (X,T*) is a fuzzy supra first category space. Then $\vee_{i=1}^{\infty}(\lambda_i)=1$,where (λ_i)'s are fuzzy supra nowhere dense sets in (X,T*). Now $\lambda_i \leq cl^*(\lambda_i)$, implies that $\vee_{i=1}^{\infty}(\lambda_i) \leq \vee_{i=1}^{\infty}cl^*(\lambda_i)$ .....(1). Also $\vee_{i=1}^{\infty}cl^*(\lambda_i)$ is a fuzzy supra F_σ-set in (X,T*). Since, (X,T*) is a fuzzy supra almost P-space, by Proposition 3.5, $cl^*(\vee_{i=1}^{\infty}cl^*(\lambda_i)) \neq 1$ .....(2). From (1), we have $cl^*(\vee_{i=1}^{\infty}(\lambda_i)) \leq cl^*(\vee_{i=1}^{\infty}cl^*(\lambda_i))$ implies that $cl^*(1) \leq cl^*(\vee_{i=1}^{\infty}cl^*(\lambda_i))$ and hence we have $1\leq cl^*((\vee_{i=1}^{\infty}cl^*(\lambda_i))$. That is, $cl^*(\vee_{i=1}^{\infty}cl^*(\lambda_i)) =1$, a contradiction to (2). Hence, we must have $\vee_{i=1}^{\infty}(\lambda_i)\neq 1$, in (X,T*) and therefore (X,T*) is a fuzzy supra second category space.

Definition 3.7

A fuzzy supra topological space (X,T*) is called a fuzzy supra Volterra space if $cl^*(\wedge_{i=1}^{N}(\lambda_i))=1$, where (λ_i)'s are fuzzy supra dense and fuzzy supra G_δ-set in (X,T*).

Proposition 3.7

If the fuzzy supra topological space (X,T*) is a fuzzy supra submaximal and fuzzy supra

hyperconnected space, then (X,T*) is a fuzzy supra Volterra space.

Proof:

Let (λ_i)'s (i = 1 to N) be fuzzy supra dense set and fuzzy supra G_δ-sets in (X,T*). Since (X,T*) is a fuzzy supra submaximal space, $cl^*(\lambda_i) = 1$, implies that $\lambda_i \in T^*$ in (X, T^*). Then we have $int^*(\lambda_i) = \lambda_i$. This implies that $cl^*int^*(\lambda_i) = cl^*(\lambda_i)$. Thus $cl^*int^*(\lambda_i) = 1$ for the fuzzy supra dense sets λ_i in (X,T*). Thus (X,T*) is a fuzzy supra strongly irresolvable space. Now $\lambda_i \in T^*$ implies that $\wedge_{i=1}^N(\lambda_i) \in T^*$. Also since (X,T*) is a fuzzy supra hyperconnected space, $\wedge_{i=1}^N(\lambda_i) \in T^*$, implies that $cl^*(\wedge_{i=1}^N(\lambda_i)) = 1$. Therefore (X,T*) is a fuzzy supra Volterra space.

Definition 3.8

A fuzzy supra topological space (X,T^*) is called a fuzzy supra weakly Volterra space if $cl^*(\wedge_{i=1}^N(\lambda_i)) \neq 0$ where (λ_i)'s are fuzzy supra dense and fuzzy supra G_δ-set in (X,T^*).

Proposition 3.8

If a fuzzy supra topological space (X,T^*) is a fuzzy supra almost GP-space, then (X,T^*) is a fuzzy supra weakly Volterra space.

Proof:

Let (λ_i)'s (i=1 to N) be fuzzy supra dense set in a fuzzy supra almost GP-space (X,T^*). Then we have $int^*(\lambda_i) \neq 0$ in (X,T^*).Then $1- int^*(\lambda_i) \neq 1$ and hence $cl^*(1-\lambda_i) \neq 1$. Now $cl^*(\vee_{i=1}^N(1-\lambda_i)) = (\vee_{i=1}^N cl^*(1-\lambda_i))$, implies that $cl^*(\vee_{i=1}^N(1-\lambda_i)) \neq 1$. Then $cl^*(1-\wedge_{i=1}^N(\lambda_i)) \neq 1$, implies that $int^*(\wedge_{i=1}^N(\lambda_i)) \neq 0$. Now $int^*(\wedge_{i=1}^N(\lambda_i)) \leq \wedge_{i=1}^N(\lambda_i) < cl^*(\wedge_{i=1}^N(\lambda_i))$ implies that $cl^*(\wedge_{i=1}^N(\lambda_i)) \neq 0$ and hence (X,T^*) is a fuzzy supra weakly Volterra space.

4.On Fuzzy Supra Strongly Hyperconnected Spaces

Definition 4.1 [5]

In a fuzzy supra topological space (X, T^*) is called a fuzzy supra strongly hyper connected space, if the following conditions hold:

(i) if λ is a fuzzy supra dense set in (X, T^*), then λ is a fuzzy supra open set in (X,T^*) and

(ii) if λ is a fuzzy supra open set in (X,T^*), then λ is a fuzzy supra dense set in (X,T^*).

Proposition 4.1

If (X,T*) is a fuzzy supra strongly hyperconnected space, then (X,T*) is a fuzzy supra nodec, fuzzy supra hyperconnected and fuzzy supra open hereditarily irresolvable space.

Proof:

Let (X,T*) be a fuzzy supra strongly hyperconnected space. Then, by Theorem 2.2, (X,T*) is a fuzzy supra submaximal and fuzzy supra hyperconnected space. Since (X,T*) is a fuzzy supra submaximal space, by Proposition 3.4, (X,T*) is a fuzzy supra nodec and fuzzy supra open hereditarily irresolvable space. Hence, (X,T*) is a fuzzy supra nodec, fuzzy supra hyperconnected and fuzzy supra open hereditarily irresolvable space.

Proposition 4.2

In a fuzzy supra strongly hyperconnected space (X,T*),

(i) If λ is a fuzzy supra open set in (X,T*), then λ is a fuzzy supra dense set in (X,T*).

(ii) If μ is a fuzzy supra nowhere dense set in (X,T*), then μ is a fuzzy supra closed set in (X,T*).

(iii) If $int^*(\delta) = 0$, for a non–zero fuzzy set δ in (X,T*), then $int^*cl^*(\delta)=0$, in (X,T*).

Proof:

(i) Let λ be a fuzzy supra open set in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, the fuzzy supra open set λ is a fuzzy supra dense set in (X,T*).

(ii) Let μ be a fuzzy supra nowhere dense set in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, by Proposition 4.1, (X,T*) is a fuzzy supra nodec space. Hence the fuzzy supra nowhere dense set μ is a fuzzy supra closed set in (X,T*).

(iii) Let $int^*(\delta)=0$, for a non-zero fuzzy set in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, by Proposition 4.1, (X,T*) is a fuzzy supra open hereditarily irresolvable space, and hence by Proposition 3.2, $int^*(\delta)=0$ implies that $int^*cl^*(\delta)=0$, in (X,T*).

Proposition 4.3

If $int^*(\delta)=0$, for a non-zero fuzzy set δ in a fuzzy supra strongly hyperconnected space (X,T*), then 1–δ is a fuzzy supra open and fuzzy supra dense set in (X,T*).

Proof:

Let δ be a fuzzy set defined on X such that $int^*(\delta) = 0$, in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, by Proposition 4.2(iii), $int^*cl^*(\delta)=0$, in (X,T*). Then, δ is a fuzzy supra nowhere dense set and hence, by

Proposition 4.2(ii), δ is a fuzzy supra closed set in (X,T*). Then, $1-\delta$ is a fuzzy supra open set in (X,T*). Now $cl^*(1-\delta)=1-int^*(\delta)= 1-0=1$, implies that $1-\delta$ is a fuzzy supra dense set in (X,T*). Hence $(1-\delta)$ is a fuzzy supra open and fuzzy supra dense set in (X,T*).

Proposition 4.4

If $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy supra dense sets in a fuzzy supra strongly hyperconnected space (X,T*), then λ is a fuzzy supra residual set in (X,T*).

Proof:

Let $\lambda = \wedge_{i=1}^{\infty}(\lambda_i) =1$, where (λ_i)'s are fuzzy supra dense sets in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, by Theorem 2.3, $1-\lambda$ is a fuzzy supra first category set and hence λ is a fuzzy supra residual set in (X,T*).

Definition 4.2

If λ is a fuzzy supra somewhere dense set in a fuzzy supra topological space (X,T*), then $1-\lambda$ is called a fuzzy supra complement of fuzzy supra somewhere dense set in (X,T*). It is denoted as fuzzy supra cs dense set in (X,T*).

Proposition 4.5

If λ is a fuzzy supra cs dense set in a fuzzy supra strongly hyperconnected space (X,T*), then $int^*(\lambda)=0$, in (X,T*).

Proof:

Let λ be a fuzzy supra cs dense set in (X,T*). Then, $1-\lambda$ is a fuzzy supra somewhere dense set in (X,T*) and thus $int^*cl^*(1-\lambda) \neq 0$ in (X,T*). This implies that $1-cl^*int^*(\lambda) \neq 0$ and then $cl^*[int^*(\lambda)] \neq 1$ in (X,T*). Thus, $int^*(\lambda)$ is not a fuzzy supra dense set in (X,T*). Then, there exists a non - zero fuzzy supra closed set μ in (X,T*) such that $int^*(\lambda) < \mu < 1$ in (X,T*). This implies that $1-int^*(\lambda) > (1-\mu)$ and then, $cl^*[1-int^*(\lambda)] > cl^*(1-\mu)$ in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, the fuzzy supra open set $1-\mu$ is a fuzzy supra dense set in (X,T*) and hence $cl^*(1-\mu) = 1$ in (X,T*). This implies that $1 \leq cl^*[1-int^*(\lambda)]$ in (X,T*). That is, $cl^*[1-int^*(\lambda)] = 1$ in (X,T*) and then $1-int^*[int^*(\lambda)] = 1$ and thus $int^*[int^*(\lambda)] = 0$. Hence $int^*(\lambda) = 0$ (since $int^*int^*(\lambda) = int^*(\lambda)$) in (X,T*).

Proposition 4.6

If λ and μ are fuzzy supra cs dense sets in a fuzzy supra strongly hyperconnected space (X,T*), then $int^*(\lambda \vee \mu) = 0$, in (X,T*).

Proof:

Let λ and μ be fuzzy supra cs dense sets in (X,T*). Then, by Proposition 4.5, $int^*(\lambda) = 0$ and $int^*(\mu) = 0$ in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, by Theorem 2.5, $int^*(\lambda \vee \mu) = 0$, in (X,T*).

Proposition 4.7

If λ and μ are fuzzy supra cs dense sets in a fuzzy supra strongly hyperconnected space (X,T*), then λ and μ are fuzzy supra closed sets in (X,T*).

Proof:

Let λ and μ be fuzzy supra cs dense sets in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, by Proposition 4.6, $int^*(\lambda \vee \mu) = 0$, in (X,T*). Then, $1-int^*(\lambda \vee \mu) = 1$ and hence $cl^*[1-(\lambda \vee \mu)] = 1$ in (X,T*). This implies that $cl^*[(1-\lambda)\wedge(1-\mu)]=1$ in (X,T*). But $cl^*[(1-\lambda)\wedge(1-\mu)] \leq cl^*(1-\lambda) \wedge cl^*(1-\mu)$ in (X,T*). This implies that $1 \leq cl^*(1-\lambda) \wedge cl^*(1-\mu)$ in (X, T). That is, $cl^*(1-\lambda) \wedge cl^*(1-\mu) = 1$, in (X,T*). Then, $cl^*(1-\lambda) = 1$ and $cl^*(1-\mu) = 1$, in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, $1-\lambda$ and $1-\mu$ are fuzzy supra open sets in (X,T*) and then λ and μ are fuzzy supra closed sets in (X,T*).

Proposition 4.8

If λ and μ are fuzzy supra somewhere dense sets in a fuzzy supra hyperconnected space (X,T*), then λ and μ are fuzzy supra open sets in (X,T*).

Proof:

Let λ and μ be fuzzy supra somewhere dense sets in (X,T*). Then $1-\lambda$ and $1-\mu$ are fuzzy supra cs dense sets in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, by Proposition 4.7, $1-\lambda$ and $1-\mu$ are fuzzy supra closed sets in (X,T*) and hence λ and μ are fuzzy supra open sets in (X,T*).

Proposition 4.9

If λ and μ are fuzzy supra somewhere dense sets in a fuzzy supra strongly hyperconnected space (X,T*), then $\lambda \wedge \mu$ is a fuzzy supra somewhere dense set in (X,T*).

Proof:

Let λ and μ be fuzzy supra somewhere dense sets in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, by Proposition 4.8, λ and μ are non-zero fuzzy supra open sets in (X,T*). Then $\lambda \wedge \mu$ is a non-zero fuzzy supra

open set in (X,T*). Then, by Theorem 2.4, $int^*(\lambda \wedge \mu) = \lambda \wedge \mu \neq 0$, in (X,T*). Now, $int^*(\lambda \wedge \mu) \leq int^*[cl^*(\lambda \wedge \mu)]$ implies that $int^*cl^*(\lambda \wedge \mu) \neq 0$ in (X,T*) and hence $\lambda \wedge \mu$ is a fuzzy supra somewhere dense set in (X,T*).

Proposition 4.10

If λ and μ are fuzzy supra cs dense sets in a fuzzy supra strongly hyperconnected space (X,T*), then $\lambda \vee \mu$ is a fuzzy supra cs dense set in (X,T*).

Proof:

Let λ and μ be fuzzy supra cs dense sets (X,T*). Then $1 - \lambda$ and $1 - \mu$ are fuzzy supra somewhere dense sets in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, by Proposition 4.9, $(1-\lambda) \wedge (1-\mu)$ is a fuzzy supra somewhere dense set in (X,T*). Then, $1 - (\lambda \vee \mu)$ is a fuzzy supra somewhere dense set in (X,T*) and hence $\lambda \vee \mu$ is a fuzzy supra cs dense set in (X,T*).

Proposition 4.11

If λ is a fuzzy supra somewhere dense set in a fuzzy supra strongly hyperconnected space (X,T*), then λ is a fuzzy supra dense set in (X,T*).

Proof:

Let λ be a fuzzy supra somewhere dense set in (X,T*). Then $int^*[cl^*(\lambda)] \neq 0$ in (X,T*). Then, $1 - int^*[cl^*(\lambda)] \neq 1$ in (X,T*). This implies that $cl^*int^*(1-\lambda) \neq 1$ in (X,T*). Then, $int^*(1-\lambda)$ is not a fuzzy supra dense set in (X,T*) and hence there exists a non-zero fuzzy supra closed set μ in (X,T*) such that $int^*(1-\lambda) < \mu < 1$ in (X,T*). That is., $1 - cl^*(\lambda) < \mu < 1$, in (X,T*). This implies that $1 - \mu \leq cl^*(\lambda)$, in (X,T*). Then $cl^*(1 - \mu) \leq cl^*[cl^*(\lambda)] = cl^*(\lambda)$, in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, the fuzzy supra open set $1 - \mu$ is a fuzzy supra dense set in (X,T*) and hence $cl^*(1-\mu) = 1$ in (X,T*). This implies that $1 \leq cl^*(\lambda)$ in (X,T*). That is, $cl^*(\lambda) = 1$ in (X,T*). Hence λ is a fuzzy supra dense set in (X,T*).

Proposition 4.12

In a fuzzy supra strongly hyperconnected space (X,T*), each non-zero fuzzy set is either a fuzzy supra nowhere dense set or fuzzy supra dense set in (X,T*).

Proof:

Let λ be a non-zero fuzzy set in (X,T*). Then, $cl^*(\lambda)$ is a non-zero fuzzy supra closed set in (X,T*). Then, either $int^*[cl^*(\lambda)] = 0$ or $int^*[cl^*(\lambda)] \neq 0$, in (X,T*).

(i) If $int^*[cl^*(\lambda)] = 0$ in (X,T*), then λ is a fuzzy supra nowhere dense set in (X,T*).

(ii) If $int^*[cl^*(\lambda)] \neq 0$ in (X,T*), then λ is a fuzzy supra somewhere dense set in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, by Proposition 4.11, the fuzzy supra somewhere dense set λ is a fuzzy supra dense set in (X,T*).

Proposition 4.13

If λ is a fuzzy supra first category set in (X,T*) such that $cl^*(\lambda) = 1$ in a fuzzy supra strongly hyperconnected space (X,T*), then there exists a fuzzy supra nowhere dense set μ in (X,T*) such that $\mu \leq 1 - \lambda$.

Proof:

Let λ be a fuzzy supra first category set in (X,T*) such that $cl^*(\lambda) = 1$ in (X,T*). Then, $\lambda = \vee_{i=1}^{\infty}(\lambda_i)=1$,where (λ_i)'s are fuzzy supra nowhere dense sets in (X,T*). Now $1-cl^*(\lambda_i)$ is a fuzzy supra open set in (X,T*). Let $\mu = \wedge_{i=1}^{\infty}[1- cl^*(\lambda_i)]$. Then μ is a fuzzy supra G_δ-set in (X,T*). Now, $\mu = \wedge_{i=1}^{\infty}[1 - cl^*(\lambda_i)] = 1-\vee_{i=1}^{\infty} cl^*(\lambda_i) \leq 1-\vee_{i=1}^{\infty} (\lambda_i) =1-\lambda$, in (X,T*). That is., $\mu \leq 1-\lambda$ in (X,T*). Then, $int^*(\mu) \leq int^*(1- \lambda)$ implies that $int^*(\mu) \leq 1 - cl^*(\lambda) =1 - 1 =0$. That is., $int^*(\mu)=0$ in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space, by Proposition 4.2(iii), $int^*[cl^*(\mu)] = 0$, in (X,T*). Hence, there exists a fuzzy supra nowhere dense set μ in (X,T*) such that $\mu \leq 1 - \lambda$.

Proposition 4.14

If $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $int^*(\lambda_i)=0$ in a fuzzy supra strongly hyperconnected space (X,T*), then λ is a fuzzy supra first category set in (X,T*).

Proof:

Suppose that $\lambda = \vee_{i=1}^{\infty}(\lambda_i)=1$, where $int^*(\lambda_i)=0$ in (X,T*). Since (X,T*) is a fuzzy supra strongly hyperconnected space by Proposition4.2(iii), $int^*[cl^*(\lambda_i)]=0$ in (X,T*) and hence (λ_i)'s are fuzzy supra nowhere dense sets in (X,T*). Hence $\lambda =\vee_{i=1}^{\infty}(\lambda_i)=1$, where (λ_i)'s are fuzzy supra nowhere dense set in (X,T*), implies that λ is a fuzzy supra first category set in (X,T*).

5. Fuzzy Supra Strongly Hyperconnected Spaces and Fuzzy Supra Baire Spaces

Definition 5.1

A fuzzy supra topological space (X,T*) is called a fuzzy supra resolvable space if there exists a

fuzzy supra dense set λ in (X,T^*) such that $cl^*(1 - \lambda) = 1$. Otherwise (X,T^*) is called a fuzzy supra irresolvable space.

Definition 5.2

A fuzzy supra topological space (X,T^*) is called a fuzzy supra almost resolvable space if $\vee_{i=1}^{\infty}(\lambda_i)=1$, where the fuzzy sets (λ_i)'s in (X,T^*) are such that $int^*(\lambda_i)=0$. Otherwise (X,T^*) is called a fuzzy supra almost irresolvable space.

Definition 5.3

A fuzzy supra topological space (X,T^*) is called a fuzzy supra extra resolvable space, if whenever λ_i and λ_j $(i \neq j)$ are fuzzy supra dense sets in (X,T^*), then $\lambda_i \wedge \lambda_j$ is a fuzzy supra nowhere dense set in (X,T^*). That is., (X,T^*) is a fuzzy supra extra resolvable if $int^*[cl^*(\lambda_i \wedge \lambda_j)]=0$, $(i \neq j)$ where $cl^*(\lambda_i)= 1$ and $cl^*(\lambda_j) =1$, in (X,T^*).

Proposition 5.1

If (X,T^*) is a fuzzy supra strongly hyperconnected space, then (X,T^*) is a fuzzy supra irresolvable space.

Proof:

Let δ $(\neq 1)$ be a fuzzy set defined on X such that $int^*(\delta)=0$ in (X,T^*). Then, $cl^*(1-\delta) = 1-int^*(\delta) = 1 - 0 = 1$. Hence $1 - \delta$ is a fuzzy supra dense set in (X,T^*). Since (X,T^*) is a fuzzy supra strongly hyperconnected space, the fuzzy supra dense set $1 - \delta$ is a fuzzy supra open set and then δ is a fuzzy supra closed set and $cl^*(\delta) = \delta \neq 1$, in (X,T^*). Hence (X,T^*) is a fuzzy supra irresolvable space.

Proposition 5.2

If (X,T^*) is a fuzzy supra almost resolvable and fuzzy supra strongly hyperconnected space, then (X,T^*) is a fuzzy supra first category space.

Proof:

Let $(X\ T^*)$ be a fuzzy supra almost resolvable space. Then $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where (λ_i)'s are fuzzy sets defined on X such that $int^*(\lambda_i) = 0$ in (X,T^*). Since (X,T^*) is a fuzzy supra strongly hyperconnected space, by Proposition 4.2(iii), that $int^*[cl^*(\lambda_i)] = 0$ in (X,T^*) and hence (λ_i)'s are fuzzy supra nowhere dense sets in (X,T^*). Thus $\vee_{i=1}^{\infty}(\lambda_i)=1$, where (λ_i)'s are fuzzy supra nowhere dense sets in (X,T^*), implies that (X,T^*) is a fuzzy supra first category space.

Proposition 5.3

If (X,T^*) is a fuzzy supra almost resolvable and fuzzy supra strongly hyperconnected space, then (X,T^*) is not a fuzzy supra Baire space.

Proof:

Let (X,T^*) be a fuzzy supra almost resolvable and fuzzy supra strongly hyperconnected space. Then, by Proposition 5.2, (X,T^*) is a fuzzy supra first category space. Then $\vee_{i=1}^{\infty}(\lambda_i)=1$, where (λ_i)'s are fuzzy supra nowhere dense sets in (X,T^*). Then, $int^*[\vee_{i=1}^{\infty}(\lambda_i)]=1=int^*(1) = 1 \neq 0$ and hence (X,T^*) is not a fuzzy supra Baire space.

Proposition 5.4

If each non-zero fuzzy supra G_δ-set is a fuzzy supra dense set in a fuzzy supra strongly hyperconnected space (X,T^*), then

(i) (X,T^*) is a fuzzy supra P-space.
(ii) (X,T^*) is a fuzzy supra almost P-space.
(iii) (X,T^*) is a fuzzy supra second category space.

Proof:

(i). Let λ be a fuzzy supra G_δ-set in (X,T^*) such that $cl^*(\lambda)=1$. Since (X,T^*) is a fuzzy supra strongly hyperconnected space, by Theorem 2.2, the fuzzy supra dense set is a fuzzy supra open set in (X,T^*) and hence (X,T^*) is a fuzzy supra P-space.

(ii). The proof follows from (i), and from the fact that each fuzzy supra P-space is a fuzzy supra almost P-space.

(iii). Let λ be a fuzzy supra G_δ-set in (X,T^*) such that $cl^*(\lambda)=1$ in (X,T^*). Since (X,T^*) is a fuzzy supra strongly hyperconnected space, by (ii), (X,T^*) is a fuzzy supra almost P-space. Then, by Proposition 3.6, (X,T^*) is a fuzzy supra second category space.

Proposition 5.5

If a fuzzy supra topological space (X,T^*) is a fuzzy supra strongly hyperconnected space, then (X,T^*) is a fuzzy supra almost GP-space.

Proof:

Let λ be a non–zero fuzzy supra dense and fuzzy supra G_δ- set in (X,T^*). Since (X,T^*) is a fuzzy supra strongly hyperconnected space, the fuzzy supra dense set is a fuzzy supra open set in (X,T^*) and hence $int^*(\lambda)=\lambda\neq 0$, in (X,T^*) and hence (X,T^*) is a fuzzy supra almost GP-space.

Proposition 5.6

If a fuzzy supra topological space (X,T^*) is a fuzzy supra strongly hyperconnected space, then (X,T^*) is a fuzzy supra weakly Volterra space.

Proof:

The proof follows from Proposition 5.5, and Proposition 3.8.

Proposition 5.7

If λ is a fuzzy supra residual set in a fuzzy supra P-space (X,T*), then λ is a fuzzy supra somewhere dense set in (X,T*).

Proof:

Let λ be a fuzzy supra residual set in a fuzzy supra P-space (X,T*). That is., $\lambda = 1 - \vee^{\infty}_{i=1}(\lambda_i)$. Now $int^*cl^*(\lambda) = int^*cl^*(\wedge^{\infty}_{i=1}(\lambda_i)) \neq 0$. This implies that $int^*cl^*(\lambda) \neq 0$ in (X,T*). Therefore λ is a fuzzy supra somewhere dense set in (X,T*).

Proposition 5.8

If (X,T*) is a fuzzy supra strongly hyperconnected space and fuzzy supra P-space, then (X,T*) is a fuzzy supra Baire space.

Proof:

Let λ be a fuzzy supra residual set in (X,T*). Then, by Proposition 5.7, λ is a fuzzy supra dense set in (X,T*). Then, by Theorem 2.1, (X,T*) is a fuzzy supra Baire space.

Proposition 5.9

If each non-zero fuzzy supra G_δ- set is a fuzzy supra dense set in a fuzzy supra strongly hyperconnected space (X,T*), then (X,T*) is a fuzzy supra Baire space.

Proof:

Let (X,T*) be a fuzzy supra strongly hyperconnected space in which each non-zero fuzzy supra G_δ-set is a fuzzy supra dense set in (X,T*). Then, by Proposition 5.4, (X,T*) is a fuzzy supra P-space. Thus, (X,T*) is a fuzzy supra strongly hyperconnected and fuzzy supra P-space. Then, by Proposition 5.8, (X,T*) is a fuzzy supra Baire space.

CONCLUSION

In this paper, Characterizations of fuzzy supra Baire spaces and other fuzzy supra topological spaces are given. Finding relations between fuzzy supra strongly hyperconnected spaces and fuzzy supra Baire spaces. Also, finding the conditions under which a fuzzy supra strongly hyperconnected spaces to became a fuzzy supra Baire spaces.

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